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Structural Design Optimization Status and Direction

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This paper discusses the use of optimization techniques for structural design. This discussion begins with a brief historical review of this technology from the initial concept to modern approximation techniques. The present state of the art is reviewed and some examples are offered to demonstrate the state of the art. Finally, future needs are addressed to indicate some of the challenges that lie ahead. It is concluded that the state of the art is now reasonably well refined. The challenge now is to assimilate this technology into the practicing design environment.

Introduction

STRUCTURAL optimization has been a topic of interest for over 100 years, beginning with the early works of Maxwell[1] and Michell.[2] In the 1940s and early 1950s, considerable analytical work was done on component optimization as represented by such works as Shanley's *Weight-Strength Analysis of Aircraft Structures*.[3]

Development of linear programming techniques by Dantzig,[4] together with the advent of the digital computer, led to the application of mathematical programming techniques to the plastic design of beam and frame structures as described by Heyman.[5]

Schmit[6] was the first to offer a comprehensive statement of the use of mathematical programming techniques to solve the nonlinear, inequality constrained problem of designing elastic structures under a multiplicity of loading conditions. He combined numerical optimization with finite element analysis, itself an emerging technology, to solve the structural synthesis problem.

Numerical optimization solves the general problem: find the set of design variables $\boldsymbol{X}$ that will[7]

$$\text{Minimize } F(\boldsymbol{X}) \tag{1}$$

Subject to

$$g_j(\boldsymbol{X}) \leq 0, \qquad j = 1, m \tag{2}$$

$$X_i^L \leq X_i \leq X_i^U, \qquad i = 1, n \tag{3}$$

The function $F(\boldsymbol{X})$ is referred to as the objective or merit function, and is dependent on the values of the design variables $\boldsymbol{X}$, which themselves include member dimensions or shape variables of a structure as examples. The limits on the design variables, given in Eq. (3), are referred to as side constraints and are used simply to limit the region of search for the optimum. For example, it would not make sense to allow the thickness of a structural element to take on a negative value. Thus, the lower bounds are set to a reasonable minimum gauge size. If we wish to maximize $F(\boldsymbol{X})$, e.g., maximize the fundamental frequency of a structure, we simply minimize the negative of $F(\boldsymbol{X})$.

The $g_j(\boldsymbol{X})$ are referred to as constraints, and they provide bounds on various response quantities. The most common constraint is the limits imposed on stresses at various points within the structure. Then, if $\bar{\sigma}$ is the upper bound allowed on stress, the constraint function would be written, in normalized form, as

$$(\sigma_{ijk}/\bar{\sigma}) - 1 \leq 0 \tag{4}$$

where i = element, j = stress component, and k = load condition.

Schmit[6] recognized that this form of defining the design problem was precisely what was needed to find the minimum weight of structures, where finite element analysis is used to calculate the needed responses.

The finite element method was now available to solve for nodal displacements in the structure using the familiar relationship

$$[\boldsymbol{K}]\boldsymbol{u} = \boldsymbol{P} \tag{5}$$

Once the displacements are calculated, the stresses in the members are recovered from those. Schmit[6] solved the minimum weight problem for the now classical three-bar truss, and showed that the optimum design was not fully stressed.

This pioneering work led to a great deal of research in the early 1960s and beyond, and it was soon recognized that gradient-based optimization methods were most efficient for solution of the optimization task. Because Eq. (5) requires a solution for the displacements as implicit functions of the design variables (contained in the stiffness matrix $[\boldsymbol{K}]$), gradients of the stresses and displacements were calculated by finite difference.

Fox[8] noted this implicit relationship during his doctoral thesis defense as justification for using finite difference gradients. It was asked if Eq. (5) could be differentiated to get

$$\frac{\partial \boldsymbol{u}}{\partial X_i} = [\boldsymbol{K}]^{-1}\left\{\frac{\partial \boldsymbol{P}}{\partial X_i} - \frac{\partial \boldsymbol{K}}{\partial X_i}\boldsymbol{u}\right\} \tag{6}$$

Because $[\boldsymbol{K}]$ has already been decomposed, the gradient of static loads is zero (for static loads) and the gradient of the stiffness matrix with respect to the design variables (at least for truss cross-sectional areas) is easily calculated and the needed information is readily available.

Since that time, gradients have been routinely calculated analytically by a variety of techniques,[9] though the original paper by Fox describing this concept is seldom referenced.

The remainder of the decade of the 1960s saw extensive research in structural optimization, but virtually no practical applications. Indeed, by the end of the 1960s it was apparent that design problems were limited to perhaps 10 variables, and

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even these tasks often required over 100 finite element analyses. Given the slowness of computers (by today's standards), together with the continually increasing size of the finite element models, it was clear that this technology was reaching a dead end. This observation was dramatically offered by Gallatly et al.,[10] when they called the 1960s the "Period of Triumph and Tragedy" for structural optimization. Furthermore, discretized optimality criteria methods presented by Venkayya,[11] based on an earlier analytical work by Prager and Taylor,[12] offered an efficient method for the solution of problems with large numbers of design variables. Optimality criterion methods were shown to be efficient for large problems, but were limited in the number of constraints that could be simultaneously considered.

Up to this time, optimization was viewed as a simple coupling of finite element analysis, sensitivity analysis, and optimization, as shown in Fig. 1.

As seen here, whenever the optimizer required evaluation of the objective function and constraints, a full finite element analysis was performed. Even using modern optimization algorithms, this process requires on the order of 50 analyses and 10 sets of sensitivity calculations. Now, if we imagine a finite element model with 500,000 degrees of freedom, 20 load cases, and requiring static and normal modes analysis, the cost of optimization is clearly prohibitive. On the other hand, if we can limit the number of detailed finite element analyses to the order of 10, then we can argue that an optimum design can be found for the cost of just finding an acceptable design using traditional *cut and try* methods.

The use of mathematical programming for structural optimization breathed new life in 1974 when Schmit and Farshi[13] published the concept of approximation techniques for structural synthesis.[13] These concepts were refined and described in more detail by Schmit and Miura.[14]

The basic concept of approximation techniques can be understood with reference to Fig. 2, which shows a simple rod in tension.

Here we approximate the stress in the element as a linear function of the *reciprocal* of the original design variable, A. This is a much higher-quality approximation than we would have by simply linearizing with respect to A.

Stress

$$\sigma = F/A \qquad \text{Nonlinear, Implicit} \tag{7}$$

Let $X = 1/A$. Now

$$\sigma = FX \qquad \text{More Linear} \tag{8}$$

$$\sigma = \sigma^0 + \nabla\sigma\delta X \qquad \text{Linear, Explicit} \tag{9}$$

The idea is that we create an approximation to the key response based on the physics of the problem at hand. We then use this approximation during the optimization phase, instead of calling the finite element analysis whenever we need to calculate the stress. Now, the objective function, being $A*L$, becomes L/X, which is clearly nonlinear. However, the objective function is easily calculated, along with its sensitivities.

Figure 3 shows a large marine gear housing that is designed using this approach. The objective was to minimize weight,

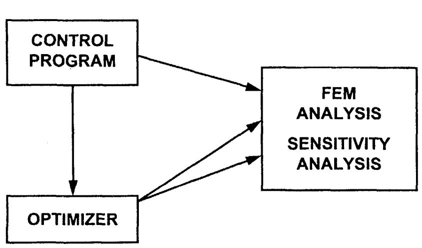

Fig. 1 Direct coupling of analysis and optimization.

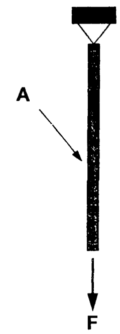

Fig. 2 Simple rod.

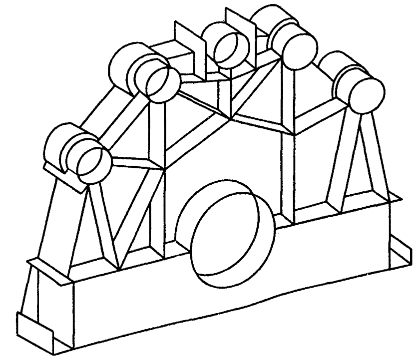

Fig. 3 Marine gear housing.

subjection to stress, and displacement constraints. Thirty sizing design variables were considered and there were 5620 inequality constraints. Six independent load conditions were considered. A one-fourth model of the doubly symmetric structure was modeled with 1623 finite elements and 7239 independent degrees of freedom. Reciprocals of the plate and rod member sizes were used as intermediate design variables. Intermediate responses (discussed later) were not used. This example was solved over 10 years ago by coupling a general-purpose optimizer[15] to Version 63 of MSC/NASTRAN.[16] The details of this problem are given in Refs. 17 and 18. Six detailed finite element analyses were required to reach the optimum.

Fleury and Sanders[19] recognized that there is a direct relationship between the optimality criteria methods of Ref. 11 and mathematical programming, noting that optimality criteria can be viewed as mathematical programming in dual space.[19] This further established the validity of approximation concepts as tools for large problems, so that now numerical optimization could solve rather large design problems while retaining the generality inherent to the method.

Using approximations, it is not necessary to approximate all response quantities considered in the optimization process. We only need to approximate those constraints that are critical or near critical for this step in the optimization process. This is referred to as *constraint screening* or *constraint deletion*. This is depicted in Fig. 4. For example, we begin by deleting from present consideration all constraints that are more negative (satisfied) than -0.3. The constraints retained after this process are identified with an $\boldsymbol{X}$ in Fig. 4. Next, we consider regions in the structure. If we have a very fine finite element model, many elements in a small region of the structure will have nearly the same stress. It is not necessary to retain all of these stresses in our approximation because the most critical of them will be representative of the responses in that region. Therefore, we retain only a subset of these constraints, represented by the character $\boldsymbol{Y}$ in Fig. 4. The result is that, while we may have over one million stress constraints in the structure, we need to retain only a small fraction of these for sensitivity and approximate optimization. It should be noted that constraint

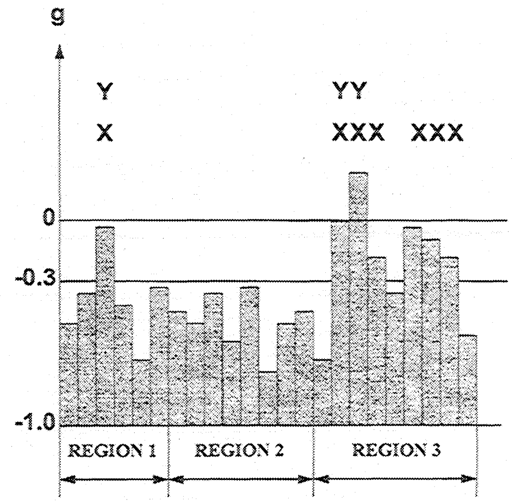

Fig. 4 Constraint deletion.

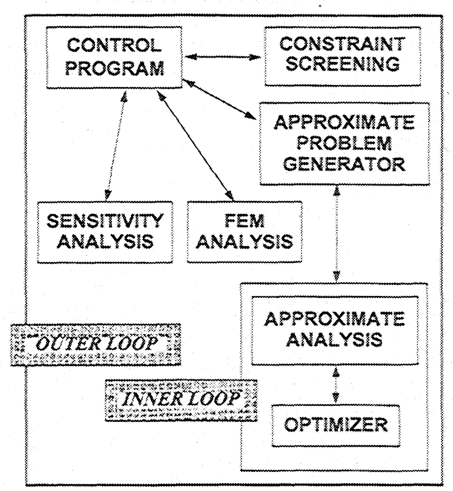

Fig. 5 Optimization using approximation techniques.

deletion is only used to reduce the computational effort needed for optimization. It does not have any effect on the final optimum achieved.

Now, instead of directly coupling the optimizer to the finite element analysis, as in Fig. 1, we create a much more sophisticated program structure, as shown in Fig. 5.

The result of this simple concept is that we could now optimize structures using, typically, as few as 10 detailed finite element analyses, even for large numbers of design variables. Indeed, for statically determinate structures, this approximation for stress (or displacement) is precise, so that only one detailed finite element analysis is needed to reach the optimum. The optimizer may still require a large number of function evaluations, but these are explicit and very cheap to evaluate.

The original approximations depicted in Fig. 2 are quite good for problems where the element stiffness matrix is a product of the original design variable (A for rods, t for membranes) and a geometric matrix. However, this approximation is not as good for cases such as beam elements or for shape optimization.

Consider the case of designing beam structures. If we approximate the responses with respect to the reciprocals of the section properties, A, I_{xx}, I_{yy}, etc., we can achieve a reasonable result. However, as a postprocessing operation, we must determine the actual structural dimensions that can be manufactured. This was attempted during the late 1970s with limited success. The difficulty is that the process may produce $A = 100$ and $I_{xx} = 1$ as a proposed optimum. However, it may be physically impossible to create a cross section with these properties.

The difficulties in applying approximation techniques to general responses was somewhat overcome with the use of conservative approximations. This concept was first proposed by Starnes and Haftka,[20] and later refined by Fleury and Braibant.[21] The basic concept is that we wish to create a conservative approximation to the response in question.

First consider a direct (linear) approximation to a constraint. This is given as

$$g_d(\boldsymbol{X}) \approx g(\boldsymbol{X}^0) + \sum_{i=1}^{N} \frac{\partial g(\boldsymbol{X}^0)}{\partial X_i}(X_i - X_i^0) \tag{10}$$

Alternatively, we may use a reciprocal approximation as presented in Ref. 13 for sizing problems. That is, we approximate the constraint in terms of the reciprocal of the design variables as

$$g_r(\boldsymbol{X}) \approx g(\boldsymbol{X}^0) - \sum_{i=1}^{N} \frac{\partial g\boldsymbol{X}^0}{\partial X_i}\left(\frac{1}{X_i} - \frac{1}{X_i^0}\right)(X_i^0)^2 \tag{11}$$

Neither of these approximations may be best in general. Also, when considering shape variables or using basis vectors described next, the reciprocal approximation may not valid, because the design variable may approach or cross zero. In the absence of better information, we may wish for the approximation to be conservative relative to a linear approximation, so that if

$$X_i \frac{\partial g(\boldsymbol{X}^0)}{\partial X_i} \geq 0 \qquad \text{Direct expansion} \tag{12}$$

if

$$X_i \frac{\partial g(\boldsymbol{X}^0)}{\partial X_i} < 0 \qquad \text{Reciprocal expansion} \tag{13}$$

This is considered to be the best approximation in the absence of better information. However, it must be remembered that this is only conservative relative to a linear approximation. It may not be conservative relative to the true function. Recognizing that a design variable may be either positive or negative (assuming the move limits allow this), we modify the simple decision in the preceding text by always using a direct expansion if X_i is near zero (see Ref. 22 for a method that attempts to overcome this by shifting the origin of the design space).

References 23–25 offer a general review of the state of the art in 1980. From these reviews, it is clear that the state of the art was reasonably well developed at that time, but very few real applications could be found. It is also clear that these authors were optimistic about the future, but that nearly 20 years later, their expectations have not been met.

Current Status of Structural Optimization

During the 1980s, and continuing today, second-generation approximation techniques began to evolve. These include the use of intermediate variables, force approximations for stress constraints, and Rayleigh quotient approximations for frequency constraints, as examples. These methods dramatically improve the quality of the approximations, but are more difficult to incorporate into existing analysis codes. For example, the use of intermediate variables and force approximations requires that, during the approximate optimization phase, the stress recovery routines be efficiently available to the optimization module of the program. Many existing analysis codes were not designed with this feature and are difficult to modify to accommodate it. Similarly, for frame structures, the input to the analysis is the section properties, yet the designer needs to change the physical dimensions during optimization.

Second-generation approximation techniques make the intermediate variables more complicated functions of the design variables, and also use intermediate responses rather than the original responses during gradient computations.[26–43] This is best understood by considering stress constraints for beam elements. Consider a simple rectangular beam element of width B and height H. These are the physical design variables that the engineer wishes to determine. Assume we have a simple stress constraint calculated at $H/2$. Then

$$\sigma = (Mc/I) \pm (P/A) \tag{14}$$

where $c = H/2$, $I = BH^3/12$, and $A = BH$ are simple, but nonlinear, functions of B and H. M is the bending moment and P is the axial force. A traditional linearization would be to create a Taylor series approximation to stress as

$$\tilde{\sigma} = \sigma^0 + \nabla\sigma \times \delta X \tag{15}$$

where $X^T = (B, H)$ and $\delta X = X - X^0$. However, it is clear that stress is highly nonlinear in the design variables B and H, and so very small move limits would be necessary during the solution of the approximate problem.

Now consider how we might better approximate the stress. First, we treat A and I as intermediate variables (called sensitivity variables in Ref. 30). Next, we calculate the gradients of M and P (intermediate responses) with respect to A and I and create a Taylor series expansion

$$\begin{Bmatrix} \tilde{M} \\ \tilde{P} \end{Bmatrix} = \begin{Bmatrix} M^0 \\ P^0 \end{Bmatrix} + \begin{bmatrix} \dfrac{\partial M^0}{\partial A} & \dfrac{\partial M^0}{\partial I} \\ \dfrac{\partial P^0}{\partial A} & \dfrac{\partial P^0}{\partial I} \end{bmatrix} \begin{Bmatrix} A - A^0 \\ I - I^0 \end{Bmatrix} \tag{16}$$

When the optimizer requires the value of stress, we first calculate A and I explicitly as functions of B and H. Then, we calculate the member end forces, M and P using the approximation given in Eq. (16). Finally, we recover the stress in the usual fashion.

With the use of such intermediate variables and responses, we achieve two important goals. First, we allow the engineer to treat the physical dimensions B and H as design variables. Second, we retain a great deal of the nonlinearity of the original problem explicitly. This allows us to make very large changes in the design variables during a given design cycle.

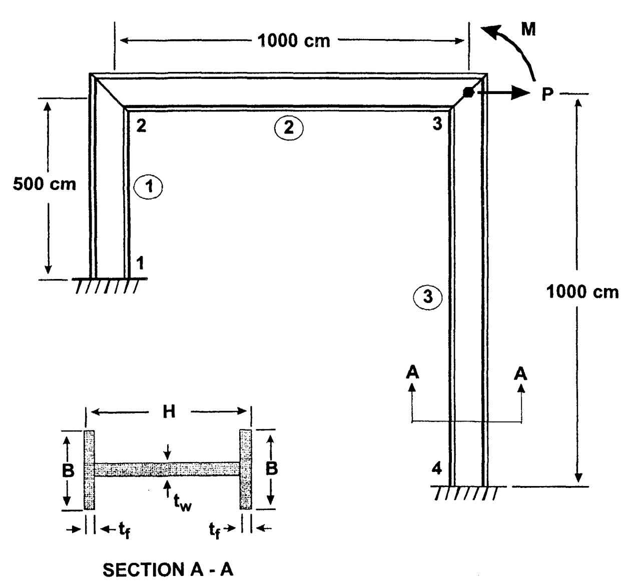

Fig. 6 Portal frame.

The portal frame shown in Fig. 6 has been used in the literature to demonstrate optimization methods.

The design task is to minimize the volume of material

$$V = \sum_{i=1}^{3} A_i L_i \tag{17}$$

Subject to stress limits in each element for each load case

$$-12{,}000 \leq \sigma_{ij} \leq 12{,}000 \tag{18}$$

Horizontal displacement limits at each joint for each load case

$$-0.4 \leq \delta_{ij} \leq 0.4 \tag{19}$$

Rotation limits at each joint for each load case

$$-0.015 \leq \theta_{ij} \leq 0.015 \tag{20}$$

Flange buckling limits for each element and load case

$$\sigma_{ab} = -1.4464E(t_f/b_f)^2 \leq \sigma_{ij} \tag{21}$$

Web shear stress limits for each element and load case

$$\tau_{ij} = (1.5|V|/t_W H) \leq 11{,}600 \tag{22}$$

This problem is solved using member section properties as intermediate variables and force approximations as intermediate responses. The actual design variables are the member dimensions. Four dimensions were designed for each element for a total of 12 design variables. The section properties are calculated using the following synthetic functions,[44] where $TW = t_w$ and $TF = t_f$:

$A(B, H, TF, TW) = 2.0*TF*B + TW*(H - 2.0*TF)$

$IZ(B, H, TF, TW) = [B*H**3$
$- (B - TW)*(H - 2.0*TF)**3]/12$

The local buckling and shear-stress constraints utilize the following synthetic functions:

$G1(B, TW, E, S) = [-1.4464*E*(TW/B)**2 - S]/5000 \leq 0$

$G2(H, TW, V) = 1.5*ABS(V)/(H*TW) \leq 11{,}600$

where $TW = t_w$ and $TF = t_f$.

This problem (as well as the remaining examples contained herein) was solved twice using the GENESIS structural optimization software,[45] beginning from a very feasible and a very infeasible design. In the worst case, only eight detailed finite element analyses were needed to reach the optimum.

While it is not apparent that force approximations are applicable to shape or configuration optimization, these methods have been shown to be applicable here as well. The reason is that, while the approximation does not become exact for statically determinate structures, it significantly uncouples it.

Figure 7 shows the initial and final configuration for the weight minimization of an 18-bar truss.

This structure was designed subject to stress and local buckling constraints. The details of the design problem are given in Ref. 34. The design variables are both member sizes and node locations. There were four member-sizing variables, being the cross-sectional areas of the top and bottom, vertical, and diagonal members. All bars in each group were linked to have the same dimension. Eight shape variables were consid-

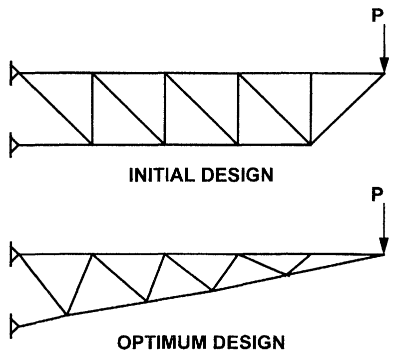

Fig. 7 Eighteen-bar truss.

ered, being the vertical and horizontal positions of the four nodes along the bottom, excluding the support node. Member forces were used as intermediate responses. An optimum of 4505 was found using eight detailed analyses. The best previous solution reported[46] was 4668, requiring 111 detailed analyses.

In a similar way, other responses can be approximated using our insight into the mathematical nature of the particular response. For example, eigenvalues may be approximated by what is called the *Rayleigh quotient approximation*, as proposed by Canfield[33];

$$\lambda_k = U/T = \Phi_k^T K \Phi_k / \Phi_k^T M \Phi_k \tag{23}$$

We now create a linear approximation to the numerator U and denominator T, independently, and use the result to estimate λ_k. Note that we can use intermediate variables, such as section properties to create these approximations, just as for stress in the previous example.

Using modern methods, member sizing and shape optimization can be simultaneously performed. For shape optimization, it is not desirable to treat each grid location as an independent design variable. This will quickly lead to unrealistic shapes, as well as generating poor meshes.

Often, the designer has several good candidate designs and just wishes to refine them to achieve an optimum, or he/she has considerable insight into what the final design should be. Also, in shape optimization, we may treat the shape as a combination of specified shapes to ensure that the resulting optimum is reasonable. Finally, by providing a set of candidate designs, the number of independent design variables can often be dramatically reduced and a manufacturable structure can be produced.

The basic concept here is that we may provide several candidate designs and then find the best linear combination of these designs to achieve the overall design objective.[47–49] In the case of shape optimization, such basis vectors can be used to control internal nodes to retain a reasonable mesh in the finite element model and, thus, reduce the need for remeshing the analysis model during the optimization process.

For example, let vectors $\boldsymbol{Y}^i$ define coordinates that can be changed, where X_i are the design variables. Then the resulting shape is

$$\boldsymbol{Y} = X_1\boldsymbol{Y}^1 + X_2\boldsymbol{Y}^2 + \cdots + X_N\boldsymbol{Y}^N \tag{24}$$

In Eq. (24), the $\boldsymbol{Y}^i$ are candidate designs. Alternatively, if the $\boldsymbol{Y}^i$ represent perturbations $\delta\boldsymbol{Y}^i$, the designed shape will be defined as

$$\boldsymbol{Y} = X_1\boldsymbol{Y}^0 + X_2\delta\boldsymbol{Y}^1 + \cdots + X_N\delta\boldsymbol{Y}^{N-1} \tag{25}$$

where, typically, $X_1 = 1$ and the remaining design variables are set to zero for the initial design. This method can be thought of physically as a design scaling approach and X_1 may or may not be allowed to change.

Clearly, Eqs. (24) and (25) are equivalent with $\delta\boldsymbol{Y}^i = \boldsymbol{Y}^i - \boldsymbol{Y}^0$, though the values of the design variables now have a different significance. The choice of which approach to use is just a matter of preference.

A simple example of this approach for shape optimization is shown in Fig. 8, which shows a one-fourth model of a plate with a hole. Note that if we treat only the positions around the hole as design variables, we are limited to very small changes in the design before it is necessary to remesh the analysis model. However, by including the positions of the interior nodes in the basis vector, we can change the external shape much more before remeshing is needed.

There are various ways to create the basis vectors for use in design. Assume, for example, that the engineer has created a computer aided design (CAD) model defining the structure to be designed. From this, we would like to automatically generate the finite element model for analysis, perform the analysis and optimization, and present the results graphically and numerically. However, except for very well-defined structures, this full automation of the analysis and optimization process is not presently possible, and a variety of intermediate steps are usually employed. For example, a series of *patches* may be defined from which the finite element model is created. Beginning with these patches, the engineer can perturb the boundary and recreate the finite element model, using the same number of elements and connectivity. The difference in the grid locations between the original and perturbed shapes would define a basis vector for use in optimization [using Eq. (25)].

Alternatively, the engineer may impose a well-chosen set of loads and boundary conditions and perform an analysis to find the deformed shape. The displacement field would then define a basis vector, and this method is referred to as a *natural shape* function.[50]

The ideal would be for the engineer to simply move a control point on a CAD model to create a proposed design perturbation. From here, the finite element mesh would be automatically recreated to produce the basis vector. As noted earlier, this cannot normally be done, in general, but the technology is progressing rapidly to achieve this goal.

Whatever method is used, there are two important considerations in the relationship between the finite element model and the shape optimization process. As the shape is changed during optimization, the finite element model can deteriorate for two reasons.

First, the shape of the elements may become so distorted that the calculated responses are not reliable. Here, some form of smoothing of the mesh is required to proceed, although it may not be necessary to change the number of elements or their connectivity.

The second situation of concern is when the shape changes such as to create a stress concentration or similar anomaly that requires mesh refinement to produce accurate results.[51] If the finite element mesh is refined, a new model is generated and the optimization process must be restarted. Given the highly efficient approximation methods now available, this is a reasonable approach, because such changes usually indicate that large design changes are being made, and so continued effort is justified. Also, beginning from the previous optimum, the new optimization should converge quickly.

Whatever reason causes the need for analysis model refinement, it is important to understand that the results of the op-

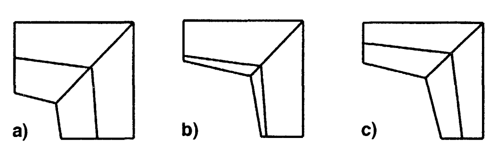

Fig. 8 Using basis shapes: a) original, b) move boundary, and c) move all nodes.

timization process are strongly dependent on the finite element model. Thus, if the mesh is refined, the stress calculated at a particular point in the structure will change, strictly as a result of the modeling (this is alleviated somewhat if *P* elements are used and only the order of the shape functions is increased, but it is still a matter of concern because care must be used in creating the model to produce reliable results). The net effect is that the optimizer sees the response as a discontinuous function and may have difficulty converging to a reliable optimum.

Experience has shown that creation of basis vectors for shape optimization and the issues of automatic mesh generation and refinement are key issues in developing reliable general-purpose shape optimization software.

Using approximation concepts, the basic program structure is shown in Fig. 5 and described as follows[14]:

1) Analyze the initial proposed design as a full finite element analysis.

2) Evaluate all constraint functions and rank them according to criticality. Retain only the critical and potentially critical constraints for further consideration during this design cycle.

3) Call the sensitivity analysis to calculate gradients of the retained set of constraints. These may be calculated as gradients of intermediate responses in terms of intermediate variables.

4) Using these gradients, construct approximations and create an optimization problem to be solved by a general-purpose optimization code, and solve it. Here, the approximation could be linear, or may be modified in various ways as described in Refs. 20–22. During this approximate optimization, move limits are imposed on the design variables to ensure the reliability of the approximation.

5) Update the analysis data and call the analysis program again to evaluate the quality of the proposed design, If the solution has converged to an acceptable optimum, terminate. Otherwise repeat from step 2.

From these five steps, it is clear that the analysis and optimization tasks are quite closely coupled. This provides the greatest possible efficiency, but at the expense of major development costs. The overall process consists of an outer loop and an inner loop, as shown in Fig. 5.

The outer loop consists of analysis, constraint deletion, gradient calculations, and the creation of the approximate problem. The inner loop consists of actually solving the approximate optimization problem. One cycle through the inner loop defines an optimization iteration, whereas one cycle through the outer loop may be called a design cycle. For each design cycle, we require a full finite element analysis and gradient computations for responses that are retained for this cycle. Typically about 10 design cycles are required, whereas perhaps 20 or more iterations are required to solve each approximate problem. Thus, the key is to create approximations of very high quality to reduce the number of design cycles (full finite element analyses) and for the approximate functions to be rapidly evaluated to reduce the cost of optimization in the inner loop.

The key concept here is to use the best approximation we can for a particular response. By doing this, we can create very high-quality approximations that allow us to change the design variables by a large amount in any design cycle. By retaining the essential nonlinearities of the problem in explicit form, the optimization process is much less dependent on move limits. Experience has shown that these second-generation approximation methods are often more efficient than previous methods and are almost always more reliable.

Whenever approximations are used in optimization, it is essential to employ move limits during the approximate optimization phase to protect against unreasonably large perturbations in the design variables or the intermediate variables. This is true no matter how good the approximation is believed to be, because in production applications, it is common that ill-conditioned design problems will be attempted.

If a simple linear approximation is used, a typical move limit may be 10% of the current value of the design variables. Using advanced approximation techniques, initial move limits of 50% are reasonable. In the case where basis vectors are used to define the design, it is common that the design variables can have zero value and they can often change sign during the optimization process. Here, special care must be taken to define reasonable move limits because a percentage change in the design variable is not physically meaningful.

Additionally, some move limit adjustment algorithm is needed to ensure overall convergence of the optimization process. This may be as simple as reducing the move limits by 50% whenever the projected and calculated values of the responses do not agree within a prescribed tolerance. In practice, experience has shown that move limits on the design variables should be individually adjusted, either by increasing or decreasing their value, to improve reliability of the process.[52,53] These more complicated algorithms are primarily designed to protect the quality and reliability of the overall process for those cases that include design variables that have significant differences in physical meaning, so that reasonable initial move limits cannot be easily prescribed.

The following examples demonstrate the application of current methods.

Consider the engine connecting rod shown in Fig. 9. We wish to minimize the material volume, subject to stress constraints. One-fourth of the symmetric structure was modeled using 1120 solid elements and approximately 5000 independent degrees of freedom. Nine design variables were used to define the shape of the structure. One load condition was considered and the mass was minimized, subject to stress constraints. This problem was solved using conservative approximations. The design required a total of 11 detailed finite element analyses.

A car body half-model finite element mesh shown in Fig. 10 is based on an existing finite element model that was created for analysis. The model consists of approximately 20,000 analysis degrees of freedom.

The design model included 67 sizing design variables and 17,500 stress constraints. Although the shape of the structure was not designed, this is a relatively poorly conditioned opti-

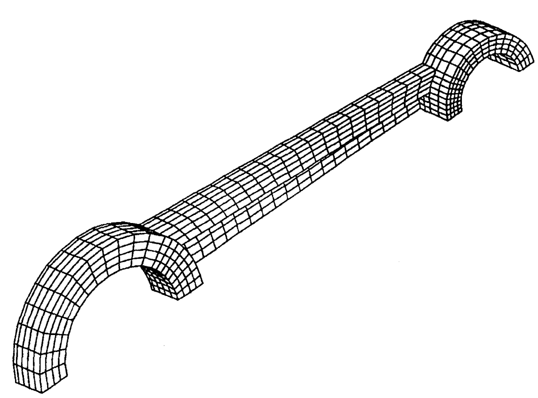

Fig. 9 Connecting rod.

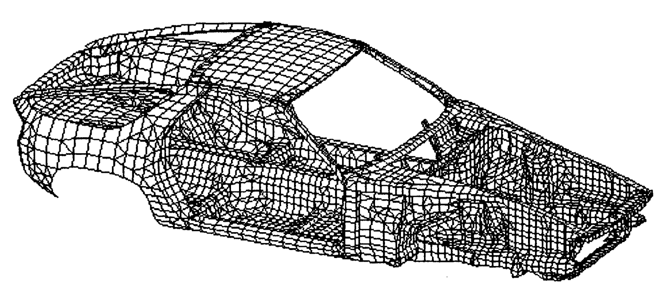

Fig. 10 Car body (analysis model courtesy Porsche AG).

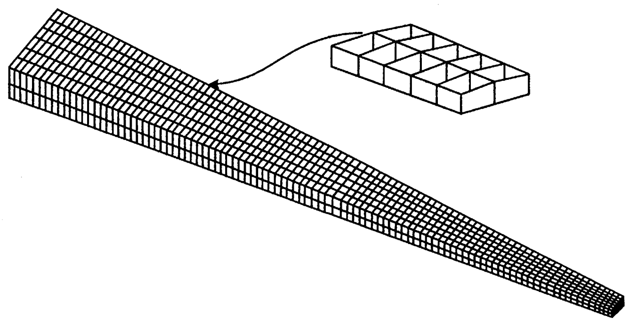

Fig. 11 Simplified wing box optimization.

mization task because an arbitrary initial design was used and because changes in the element thicknesses have a significant effect on the load paths. Despite this, the optimization process was successful in reducing the mass by 30% using seven detailed finite element analyses.

Figure 11 shows a simplified aircraft wing box designed for two independent load cases and with constraints on stresses, deflections, and eigenvalues. There were a total of 125 design variables and the analysis model consisted of about 11,000 degrees of freedom. The solution of this optimization task required seven detailed finite element analyses and took about 1 h on a 150 Mz personal computer.

While these examples are admittedly quite simple by today's standards (actual production examples are proprietary), they offer an indication of the power and reliability of modern approximation methods for structural optimization. Thanedar and Chirehdast[54] offer recent examples of sizing and shape optimization in the automotive industry.

Using current methods, design problems in excess of 2000 variables have been solved. The number of constraints can be arbitrarily large because we temporarily delete those that are not critical or near critical. Indeed, it is common in finite element based structural optimization to consider over one million nonlinear constraints.

Beginning in the mid-1980s, optimization has been added to virtually all major commercial finite element analysis programs. Some use very simple, pre-1974 methods, while others make full use of second-generation approximation techniques described here. However, commercial application of this technology is remarkably limited. The good news is that real applications of structural optimization appear to be growing at an exponential rate. The bad news is that we are on the left side of the exponential curve.

Future Direction

In 1982, this author argued that "It is perhaps sufficient to be able to claim with some certainty that there is a future for structural synthesis."[25] Today, it is clear that this future was much farther away than imagined at that time. It is now perhaps sufficient to claim that the future is here.

To make structural optimization a widespread reality, we must consider not just technical issues. Indeed, it is this author's experience that the key is human nature. Specifically, people become comfortable doing what they did last year and the year before. They (and more importantly, their management) must now be convinced that new technology can help their corporation and improve the bottom line. The evidence that structural optimization can do these things is compelling, but it remains a mystery to this author how we in the advanced technology community can convince corporations to invest in the needed training. Software costs are not relevant. Software is very cheap, a corporate license being equivalent to perhaps 2–5 person years cost. If the benefit equals 6–10 person years in revenue, then optimization is an attractive technology. Having said this, it has been this authors experience over 25 years that optimization never reduces engineering costs. There seems to be a fundamental natural law that we will use all of the time and funds available to produce a design. Therefore, the goal should be to produce a much better design in the same time/cost or, perhaps more importantly, reduce design cycle times to create a good or better design, and then move on to the next project.

Here, we will discuss the direction of structural optimization from a technical, philosophical, and political viewpoint.

In offering the ideas and opinions contained herein, many can (and surely will) argue with their validity. That is good, and can only help to advance the debate and the use of optimization. This author hopes he is wrong in some of these opinions. As a researcher in design optimization, he has made so many mistakes "it doesn't even hurt anymore."

A key development in making structural optimization a commercial reality was the addition of design sensitivity calculations to MSC/NASTRAN by the MacNeal-Schwendler Corporation in 1984.[16] Although optimization was not added for several more years, this feature gave the user the opportunity to learn the value of sensitivity information in making design decisions, and gave third parties the opportunity to couple a commercial finite element code with optimization.

References 17 and 18 demonstrate one such application, shown in Fig. 3 herein. Although MSC/NASTRAN was not the first commercial FEA program to include optimization, the widespread use of this software provided a major inducement to other vendors to add optimization.

During the 1980s, most major finite element software vendors added optimization, and at least one new product was generated to utilize the second-generation approximation methods described here.[45]

Today, two questions seem particularly relevant: how do we ensure much greater use of this technology, and what can we anticipate in the future?

First of all, the most powerful means of getting structural optimization used by a broad spectrum of designers is demonstrated successes such as are presented by Thanedar and Chirehdast[54] and White and Webb.[55] While publication of such commercial applications are necessarily sanitized to protect proprietary information, they clearly demonstrate the value of optimization in product design. Just as addition of optimization to a major finite element program encourages (indeed forces) the competition to add optimization to their products, success stories such as this provide strong motivation for industry to use this technology.

A key problem that this author has lamented for over 25 years is that most universities do not teach the fundamentals of optimization, much less structural optimization. Furthermore, when it is taught, it is usually treated as an applied mathematics course, rather than an everyday design tool. It is essential that engineering design courses include this technology as a design tool, so that graduates will be equipped to use it in their everyday work.[56]

A second problem in gaining more widespread use of optimization is that (related to the education problem) most applications are in the research or advanced design departments of corporations. These groups have a vested interest in creating an aura of complexity to this technology as they continue their research. After all, how can I justify research to improve structural optimization efficiency when commercial vendors already offer software that will find an optimum using (typically) less than 10 finite element analyses. We already have the tools to design in one computer run at a cost that is far less than the cost of achieving only an acceptable design using traditional cut and try methods requiring many days and many computer runs.

There is clearly a place for industrial research to address the unique needs of a given corporation. However, strong oversight is needed to ensure that the researchers help, rather than hinder, implementation of new technology in their respective corporations.

Recognizing that optimization is now a useful design tool for everyday applications, and having identified some needs to make this happen, we now briefly consider what can be expected in the future.

As already noted, numerous commercial structural optimization capabilities are now available, using various levels of sophistication. The providers can be expected to continue to develop and enhance these programs.

In most cases, enhancements will expand the capabilities and improve efficiency. For example, one area of efficiency improvement is the way in which sensitivities are calculated. Some codes use direct differentiation, whereas others use the adjoint method. In practice, a structural optimization program should have both, and the method used should depend on the number of design variables, load cases, and sensitivities needed.[57] In fact, as early as 1976, the research code "ACCESS1"[14] included automatic switching between the direct and adjoint methods to maximize computational efficiency, while this level of sophistication is finding its way into commercial software only now.

Of paramount importance to more widespread use of structural optimization are ease of use issues. Computer aided engineering and related graphical interface developers have been slow to add the needed design model creation capabilities to their products. As customer pressure increases, we can expect that the graphical interface developers will take optimization more seriously and provide the needed tools. This will, in turn, dramatically increase the use of optimization over a broad range of industries.

Using approximation techniques, the actual optimization process typically accounts for only a few percent of the overall computational effort. However, as structural optimization becomes more widely used, the number of design variables will continue to increase. Most modern optimization algorithms solve a subproblem for finding the search direction. If there are large numbers of variables and active constraints, this subproblem can become quite time consuming and memory intensive. Thus there is a clear need to develop algorithms that will solve such problems efficiently and reliably. The goal should be to create optimization algorithms that can handle tens of thousands of variables with an equal number of active constraints.

Topology optimization has recently received increasing interest. The goal is to begin with a block of material and use topology optimization to determine the basic configuration.[58-60] On a theoretical level, it is clear that the optimum topology will be a truss. However, most structures of interest here are shell or solid structures. Therefore, upon finding a basic configuration with topology optimization, the model must be refined to create a realistic structure.[61] Clearly, considerable research remains to be done on issues of finding a basic topology, automatically convert the result to a realistic finite element model, and refine it with available sizing and shape optimization software.

A perennial problem in structural optimization is how we deal with the existence of relative minima.[62,63] It must be noted, parenthetically, that relative minima exist whether optimization is used or not. It's just that, without optimization, we're so focused on just finding an acceptable design that we seldom think of relative minima. With the ability to rapidly obtain at least a local optimum, it is natural to now ask how we might do even better.

The relative minima problem may be addressed by several methods. Of course, we could use methods such as genetic algorithms with the full finite element analysis, but this is prohibitively costly. On the other hand, if we base the optimization on approximation techniques, the approximation may not capture sufficient nonlinearity to find the global optimum. References 64 and 65 represent recent work based on a branch and bound approach and selective restarting, respectively.

Of course, once the optimization problem has been created, we can restart the process based on engineering judgement. This is valuable for helping the engineer better understand the optimization process. On the other hand, as the power of computers continues to increase, one of the previously referenced methods, or a method yet to be devised, can be used to fully automate the process.

Finally, the technology already available can provide important insights for expanding optimization into other areas. For structural design, optimization based on material and/or geometric nonlinear analysis is a key topic. However, this technology should not be limited to structures, and the current emphasis on multidisciplinary analysis and optimization must continue to seek ways to efficiently combine many disciplines.

Summary

Structural optimization technology is now reasonably mature, and these methods have been added to most commercial finite element codes. Various methods are employed, ranging from simple coupling between the analysis and optimization to quite sophisticated use of approximation techniques.

The key issue now is to make this an everyday design tool. To achieve this, universities must begin to teach it as a design method, software vendors (particularly graphical interface providers) must dramatically improve ease of use, and corporations must invest in the needed training to make optimization a standard practice.

The combination of these efforts will dramatically reduce product development time, while at the same time improving product quality.

There is now sufficient evidence to argue that structural optimization can indeed do these things. While considerable refinement remains to be done, the tools are now available to address a very large percentage of structural design tasks. It is time to move aggressively to get structural optimization out of the research department and into the design department.

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